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VELOCITY FIELD EXCITED BY WING VIBRATIONS PROPAGATING ALONG ITS SURFACE WITH SUPERSONIC VELOCITY

by

E.A. Krasil'shchikova

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SUMMARY

The problem discussed in the present paper results in defining a function which determines the amplitude and the initial phase of oscillations at each oscillating point of the surface of the wing.

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1. Let us consider a rectilinear forward motion of a wing with constant velocity u inside a boundless volume of ideal compressible medium. Beginning from a certain moment of time t_0, small oscillations propagate along the wing's elastic surface with supersonic velocity v. The normal velocity component, due to the basic motion, is given on both sides of the wing in the form

$$v_{0n} = -u\alpha, \tag{1}$$

where α is the angle of attack of streamlined surface's elements. The normal velocity component due to vi rations is given in the form

$$v_{\Delta n} = A_{\Delta}, \tag{2}$$

Where A_{Δ} is a function of time and points of streamlined surface. Functions and A_{Δ} are small and may be arbitrary integrable functions of their arguments.

Considering that the medium is weakly disturbed, we shall consider the problem of determination of the field of velocities in linearized setup [1, 2]. We shall assume the medium's motion as irrotational and taking place in the absence of external forces. Let us take a mobile axis system of coordinates Oxyz, invariably linked with the moving wing (Fig.1). The axis Oz is directed perpendicularly to the plane of the drawing.

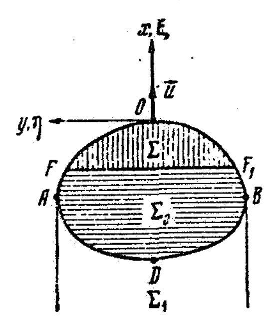

Fig.1

The potential of velocity ϕ satisfies the equation

$$(u^2 - a^2)\varphi_{xx} - a^2\varphi_{yy} - a^2\varphi_{zz} - 2u\varphi_{xt} + \varphi_{tt} = 0, \tag{3}$$

where a is the speed of sound in an unperturbed medium, and the boundary conditions in the plane xOy.

In the region Σ_1, i.e. the projection of the wing on the plane xOy ahead of the front of vibrations' propagation (line FF_1 in Fig.1), the derivative is

$$\varphi_z = -u\alpha(x, y) = A_0(x, y). \tag{4}$$

In the region Σ i.e., the projection of the wing on the plane xOy behind the fron FF_1 , the derivative is

$$\varphi_z = A_0(x, y) + A_\Delta(x, y, t) = A(x, y, t). \tag{5}$$

In the region Σ_1, i. e., the projection of the vortical shroud on the plane xOy, we have

$$\varphi_t - u\varphi_x = 0. \tag{6}$$

The potential ϕ is zero everywhere in the plane xOy outside the region $\Sigma_0 + \Sigma + \Sigma_1$; thus,

$$\varphi = 0. \tag{7}$$

Besides, the Chaplygin-Zhukovskiy's principle must be observed at the trailing edge of the wing at any moment of time.

Therefore, the problem consists in the following: to find in the half-space $z \geqslant 0$ a function $\phi(x, y, z, t)$ that would satisfy Eq.(3), the boundary conditions (4)-(5), given in regions with mobile boundary, and the boundary conditions (6) - (7), given in regions with fixed boundaries. The solution of the problem in the half-space $z < 0$ will be found from the condition $\phi(x,y, -z,t) = -\phi(x,y,z,t)$.

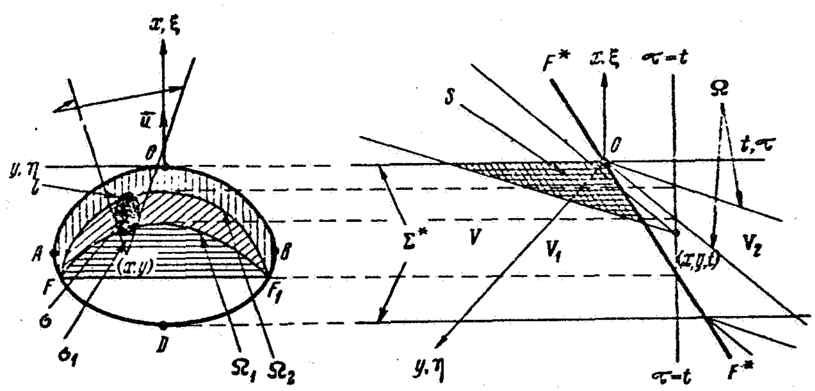

Fig.2

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2. For the solution of this pooblem we shall apply the method developed in papers [2, 4]. Let us turn to space xyt and consider in it a region V, inside of which the derivatives ϕ_z with respect to flowing around condition are given. Region V is bounded by surface Σ^*. This surface Σ^* is a cylindrical surface with genetatrices parallel to time axis 0t, and a directrix representing the contour AOBD, which is given by the equation (Fig.2)

$$\eta = \psi(\xi)$$

Let the line FF_1, - the propagation front of vibrations along the wing's surface be displaced according to the law: $x = f(t)$, whereupon $f'(t) = v$. The surface F^*, given by equation $\xi = f(\tau)$, divides the region V into two parts V_1 and V_2 with different values of the derivative ϕ_z, according to conditions (4) and (5). To the left of the surface F^*, in the region V_1 , the derivative is $\phi_z = A_0$, and to the right, in the region V_2 , it is $\phi_z = A$ (Fig.2).

Let us take the solution of Eq.(3) in the form [3]

$$\varphi(x, y, z, t) = \frac{u^2 - a^2}{2\pi} \iint\limits_{S(x,\,y,\,z,\,t)} \frac{\varphi_z\left(\xi, \eta, 0, t - \dfrac{u(x-\xi) + ar}{u^2 - a^2}\right) dS}{\sqrt{(u^2-a^2)^2 r^2 + [a(x-\xi) - ur]^2 + a^2 k^4 (y-\eta)^2}}, \tag{8}$$

$$r = \sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2 z^2}, \quad k = \sqrt{u^2/a^2 - 1},$$

where the integration region S is the surface of a hyperboloid determined by the equation

$$(x + \xi)^2 + (y - \eta)^2 + z^2 + 2u(x - \xi)(t - \tau) + (u^2 - a^2)(t - \tau)^2 = 0 \tag{9}$$

and the inequality $\tau < t$. We shall pass in formula (8) from the surface integral to double integrals with plane region of integration in the plane xOy. Let us consider a supersonic wing velocity $u > a$; then we shall obtain

$$\varphi(x, y, z, t) = -\frac{1}{2\pi} \iint\limits_{S^*(x,\,y,\,z)} \frac{\varphi_z(\xi, \eta, 0, \tau_1)}{r} d\xi\, d\eta - \\ -\frac{1}{2\pi} \iint\limits_{S^*(x,\,y,\,z)} \frac{\varphi_z(\xi, \eta, 0, \tau_2)}{r} d\xi\, d\eta. \tag{10}$$

$$\tau_1 = t + [u(x-\xi) + ar]/(u^2 - a^2), \quad \tau_2 = t + [u(x-\xi) - ar]/(u^2 - a^2).$$

Region S^* is bounded from above by a Mach wave, and from below by Mach hyperbola, or, as $z = 0$, by Mach lines.

When constructing the solution, an essential role is played by the line of intersection of surfaces S and F^*. We shall denote the projection of this line on the plane xOy by l. Curve l subdivides the plane region S^* into parts with different values of the derivative ϕ_z. If the propagation front of vibrations along the wing's surface constitutes a straight line FF_1, shifting oppositely to the motion of the wing with constant velocity $v > u + a$, the surface F^* is a plane determined by the equation $\xi + v\tau = 0$, and the curve l

is an ellipse determined by the equation

$$v^2(x-\xi)^2+v^2(y-\eta)^2+v^2z^2+ \\ +2uv(x-\xi)(vt+\xi)+(u^2-a^2)(vt+\xi)^2=0. \qquad (11)$$

Note that ellipse l is always written into the Mach hyperbola , or at $z = 0$, into the angle formed by Mach lines.

Assuming that the condition

$$|u\psi'(\xi)\,/\,\sqrt{\psi'^2(\xi)+1}|\geqslant a,$$

is fulfilled on the contour AOBD, we shall represent the solution (10) in the form

$$\varphi(x,y,z,t)=\varphi_0(x,y,z)- \\ -\frac{1}{2\pi}\iint\limits_{\sigma(x,y,z,t)+\sigma_1(x,y,z,t)}\frac{A_\Delta(\xi,\eta,\tau_1)}{r}d\xi\,d\eta-\frac{1}{2\pi}\iint\limits_{\sigma_1(x,y,z,t)}\frac{A_\Delta(\xi,\eta,\tau_2)}{r}d\xi\,d\eta, \qquad (12)$$

where the integration region σ is the part of the region S* that was found to be inside the ellipse l at the moment of time $\underline{t}$, and the region $\sigma_1$is the part of the region S* situated outside the ellipse l, below the arc of ellipse K_1LK_2, The points K_1 and K_2 are the points where ellipse l is tangent to the Mach hyperbola (Fig.3. In formula (12, function ϕ_0 is the solution of the well known problem of settled supersonic gas flow past the considered wing [5].

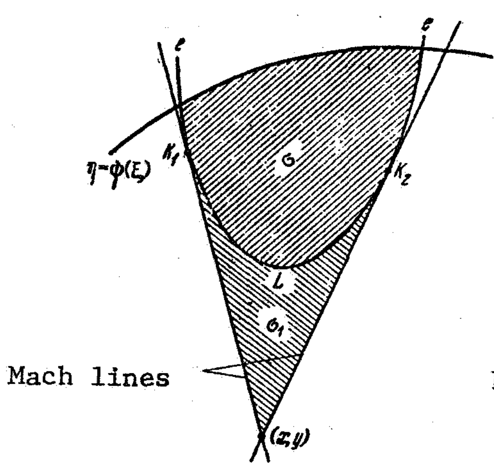

Fig.3

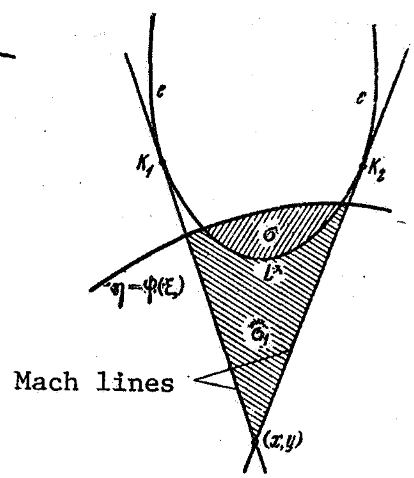

Fig.4

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3. The analytical form of problem's solution depends on the mutual disposition of ellipse l and of wing contour AOBD, which determines the integration regions σ and σ_1 in the solution (12).

Let us examine the moment of time t, belonging to the interval

$$0 = t_0 \leqslant t \leqslant t_1 = d / v,$$

where d is the length of segment OD. Let us draw a plane $\tau = t$ in the space xyt. The projection of the line of plane's $\tau = t$ intersection with the plane F^* on the plane xOy will be denoted by FF_1 (Fig.2). Let us consider in the space xyt the family of cones determined by the equation

$$(X-\xi)^2 + (Y-\eta)^2 + 2u(X-\xi)(T-\tau) + (u^2-a^2)(T-\tau)^2 = 0$$

and the inequality $\tau > T$, with summits on the line of intersection of Planes F^* and the surface Σ^*. We shall denote the envelope of this family by Ω. The projections of the line of intersection of the plane $\tau = t$ with the envelope Ω on the plane xOy will be denoted by Ω_1 and Ω_2. Let us find the equations of curves Ω_1 and Ω_2 in parametric form

$$\begin{gathered} v^2(x^*-\xi)^2 + v^2[\psi(x^*)-\eta]^2 - 2uv(x^*-\xi)(x^*+vt) + \\ +(u^2-a^2)(x^*+vt)^2 = 0, \\ v^2(x^*-\xi) + v^2[\psi(x^*)-\eta]\psi'(x^*) - uv(x^*-\xi) - \\ uv(x^*+vt) + (u^2-a^2)(x^*+vt) = 0, \end{gathered} \tag{13}$$

where x^* is a parameter.

The lines FF_1, Ω_1, Ω_2 divide the regions with different analytical character of problem's solution. To the region comprised between the straight line FF_1 and the curve Ω_1, responds the solution (12) in which the integration region σ is part of S^* bounded by ellipse l, located fully inside S^* (Fig.2). To the region comprised between curves Ω_1 and Ω_2, responds the solution (12), in which σ is part of S^*, cut by ellipse l, crossing the contour of the wing. At the same time, the points of tangency K_1 and K_2 may be located inside (Fig.3), as well as outside of the wing (Fig.4). To the region comprised between curve Ω_2 and the Mach wave, responds the solution (12) when the integration in both integrals extends over the enture region S^*, that is, region σ is absent, while region σ_1 coincides with the region S^*.

If the propagation velocity of vibrations along the streamlined surface satisfies the inequality $v < u + a$, curve l is a hyperbola.

4. In particular, when harmonic oscillations with frequency ω propagate along the elastic surface of the wing, the function

$$A_\Delta(x, y, t) = A_1(x,y)\exp i\omega(t+\alpha_1(x,y)) = \operatorname{Re} A_2(x,y)\exp i\omega t.$$

Utilizing the relation

$$\exp \frac{i\omega a}{u^2 - a^2} r + \exp - \frac{i\omega a}{u^2 - a^2} r = 2 \cos \frac{\omega a}{u^2 - a^2} r,$$

we shall represent the solution (12) in the form

$$\varphi = \varphi_0 - \frac{1}{2\pi} \operatorname{Re} \exp(i\omega t) \exp\left(\frac{i\omega u}{u^2 - a^2} x\right) \times$$
$$\times \iint_{\sigma} \frac{A_2(\xi, \eta)}{r} \exp(i\omega t) \exp\left(\frac{-i\omega u}{u^2 - a^2} \xi\right) \exp\left(\frac{i\omega a}{u^2 - a^2} r\right) d\xi\, d\eta -$$
$$- \frac{1}{2\pi} \operatorname{Re} \exp(i\omega t) \exp\left(\frac{i\omega u}{u^2 - a^2} x\right) \times$$
$$\times \iint_{\sigma_1} \frac{A_2(\xi, \eta)}{r} \exp\left(- \frac{i\omega u}{u^2 - a^2} \xi\right) \cos \frac{\omega a}{u^2 - a^2} r\, d\xi\, d\eta,$$

where the given function A_2 determines the amplitude and the initial phase of oscillations at each oscillating point on the surface of the wing.

**** T H E E N D ****

Institute of Problems
of Mechanics of the
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